Polsat: A Portfolio LTL Satisfiability Solver

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Abstract. In this paper we present a portfolio LTL-satisfiability solver, called Polsat. To achieve fast satisfiability checking for LTL formulas, the tool integrates four representative LTL solvers: pltl, TRP++, NuSMV, and Aalta. The idea of Polsat is to run the component solvers in parallel to get best overall performance; once one of the solvers terminates, it stops all other solvers. Remarkably, the Polsat solver utilizes the power of modern multi-core compute clusters. The empirical experiments show that Polsat takes advantages of it. Further, Polsat is also a testing platform for all LTL solvers.

1 Introduction

Linear Temporal Logic (LTL) satisfiability checking plays an important role in ensuring the quality of temporal specifications that are often used in an early stage in designing processes [RV10]. Temporal system requirements consist of a set of LTL properties identifying system properties that are supposed to hold in all system executions. Thus, these formulas must be satisfiable, and their conjunction must be satisfiable as well. Satisfiability checking must be scalable due to the need to handle complex temporal properties.

Earlier work [RV10] and [SD11] reported on extensive experimental investigations in LTL satisfiability checking. Rozier and Vardi reached the conclusion that when it comes to LTL satisfiability checking via reduction to model checking, the symbolic approach is superior to the explicit approach [RV10]. Nevertheless, they showed in later work that no single symbolic approach is dominant across their extensive benchmark suite [RV11]. Schuppan and Darmawan considered a wide range of solvers implementing three major classes of algorithms: reduction to model checking, tableau-based approaches, and temporal resolution [SD11]. They argued that no solver dominates across their benchmark suite. Our previous work [LZP\textsuperscript{+}13] on LTL satisfiability checking supports this conclusion further, but discovers that on-the-fly explicit approach is advantageous in checking satisfiable formulas. This motivated us to extend the portfolio approach of [RV11], but go beyond symbolic model-checking techniques and develop a portfolio LTL satisfiability solver that integrates several types of LTL satisfiability solvers and utilizes the power of modern multi-core compute clusters.
We describe here a portfolio LTL satisfiability solver, called Polsat. The tool integrates four representative LTL solvers: pltl, TRP++, NuSMV, and Aalta. The approach of Polsat is to run the solvers in parallel to get the best overall performance; once one of the solvers terminates, it stops all other solvers. To test the performance of Polsat, we collect in this paper all existing benchmarks of LTL satisfiability checking [RV10] [SD11] [LZP+13].

The empirical results show that Polsat takes advantages of the integrated solvers, and scales better for a large selection of benchmarks, especially those random formulas.

Another contribution of this paper is that Polsat provides testing platform for LTL solvers. A tool developer can use the benchmarks provided by the platform to test the solver under development and compare the results with other solvers. Thus, the tool developer can study carefully the advantage and disadvantage of the tool under development, and optimize it based on the testing results. For instance, our earlier tool, Aalta, benefited from this platform by designing new heuristics to improve tool performance.

2 Solvers

[SD11] classified three major classes of solvers based on the techniques the solvers often use: reduction to model checking, tableau-based approaches and temporal resolution. Here, we add a new class named hybrid approaches, which combines different techniques together to achieve better performance. Solvers selection strategy is discussed below.

**Reduction to model checking.** We choose NuSMV [CCG+02] as the representative. [RV10] and [SD11] carefully evaluated model checking tools such as NuSMV and ALASKA [DDMR08]. Based on their observation, we ruled out explicit state model checkers, as they did not scale comparing to symbolic ones. ALASK is not included because it fails to run on our experimental cluster platform. Thus NuSMV is chosen with both its BDD- and SAT-based approaches.
Tableau-based approaches. We choose pltl [Sch98] as the representative. From the experiments by [SD11], pltl has the best potential in this type of solvers. Our previous experiments also confirm this conclusion.

Temporal resolution. We choose TRP++ [HK03] as the representative. From the observations of [SD11], TRP++ dominates most of cases in this type of solvers.

Hybrid approaches. We choose Aalta [LZP+13] as the representative. This type of solvers includes PANDA [RV11] and Aalta [LZP+13]. PANDA tool is basically a model checking based approach but integrates multiple novel encodings of symbolic transition-based Büchi automata. Aalta belongs to the tableau-based approach but integrates some interesting heuristics. Our previous study showed that Aalta has a best potential in most cases compared to PANDA.

Summarizing, Polsat tool integrates solvers including NuSMV, pltl, TRP++, and Aalta. Since NuSMV provides both the BDD-based and SAT-based model checking, we integrated both two functionalities in Polsat respectively.

3 The framework of Polsat

A general framework of Polsat is shown in Fig. 1: it consists of three components, that are, the input, solver set and output module. Details for each component are specified in the following.

As soon as Polsat is invoked, it creates five threads to run these solvers – each solver occupies one unique thread. Once one of the solvers finishes checking then the corresponding thread will kill all other threads, which is illustrated in the figure, as all solvers can communicate through the bus. After that, the remaining thread will send the solver’s results to the Output module for further processing.

One of main Polsat’s features is, it also supports to integrate external solvers in addition to those have been integrated – with the only restriction that the solver has to provide the same input and output interface as Polsat. Using the parameter -add solverpath, one can import an external solver whose path is located in solverpath. This feature makes Polsat extensible, and provides testing platform for LTL solvers.

3.1 Input

Polsat supports the standard LTL syntax, that is, an LTL formula $\varphi$ is defined recursively as:

$$\varphi := \text{true} \mid \text{false} \mid p \mid \neg \varphi \mid \varphi \land \varphi \mid \varphi \lor \varphi \mid X\varphi \mid \varphi U \varphi.$$  

Also, we can introduce the operator $R$ (release), which is the dual operator of $U$ (until): $\varphi_1 R \varphi_2 \equiv \neg(\neg \varphi_1 U \neg \varphi_2)$. Specially, the $G$ (Global) and $F$ (Future) operators are interpreted as $G\varphi \equiv \text{false} R \varphi$ and $F\varphi \equiv \text{true} U \varphi$. As the same in propositional logic, it still holds that $\varphi_1 \rightarrow \varphi_2 \equiv \neg \varphi_1 \lor \varphi_2$ and $\varphi_1 \leftrightarrow \varphi_2 \equiv (\neg \varphi_1 \lor \varphi_2) \land (\varphi_1 \lor \neg \varphi_2)$ for LTL formulas. Among the operators above, Polsat recognizes the alternative symbols. The explicit representing is shown in Table 2.
Polsat has integrated several off-the-shelf solvers, and these solvers may have different input formats. To successfully invoke these solvers, the Parser module also integrates internal translators from the input of Polsat to those of them.

### 3.2 Output

The output of Polsat includes the following information: the checking result (“sat” or “unsat”), the solver where the result comes from, and the execution eclipse time. As the outputs vary on the different solvers, the Output module shown in Fig. 1 is designed to unify the outputs from different integrated solvers.

### 4 Empirical Experiments

We conducted all the experiments on SUG@R cluster\(^5\). SUG@R is comprised of 134 Sun Microsystems SunFire x4150 nodes, each of which contains two quad-core 2.83GHz Intel Xeon Harpertown CPUs with 16GB RAM.

The benchmarks we used are mainly from \[SD11\]. We call the benchmarks Schuppan-collected for convenience. To check the scalability of LTL solvers, we also tested the random conjunction formulas proposed in \[LZP^+13\]. A random conjunction formula has the form of \(\bigwedge_{1 \leq i \leq n} P_i\), where \(P_i\) is a random specification pattern\(^6\). In our experiments, the timeout for every testing formula is 60 seconds. Note the time is also counted if the running time of a formula checking reaches the timeout.

The experimental results on Schuppan-collected benchmarks are shown in Table 1. The first row lists all the types of this benchmark and the second to

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Table 1. Comparison results for the Schuppan-collected benchmarks

<table>
<thead>
<tr>
<th>Formula Type</th>
<th>pltl</th>
<th>TRP++</th>
<th>NuSMV</th>
<th>NuSMV-BMC</th>
<th>Aalta</th>
<th>Polsat</th>
</tr>
</thead>
<tbody>
<tr>
<td>/acacia/demo-v3</td>
<td>366.805</td>
<td>5.958</td>
<td>2753.55</td>
<td>1.004</td>
<td>557.862</td>
<td>4.326</td>
</tr>
<tr>
<td>/alaska/lift</td>
<td>5800.595</td>
<td>14989.337</td>
<td>13478.447</td>
<td>2797.13</td>
<td>8151.248</td>
<td>2721.996</td>
</tr>
<tr>
<td>/anzu/amba</td>
<td>965.456</td>
<td>5914.278</td>
<td>6088.505</td>
<td>398.177</td>
<td>2278.774</td>
<td>410.652</td>
</tr>
<tr>
<td>/anzu/genbuf</td>
<td>2849.786</td>
<td>6609.282</td>
<td>7085.315</td>
<td>695.145</td>
<td>2405.892</td>
<td>697.07</td>
</tr>
<tr>
<td>/Rozier/counter</td>
<td>1415.379</td>
<td>1570.318</td>
<td>5639.615</td>
<td>3981.308</td>
<td>3771.958</td>
<td>1388.318</td>
</tr>
<tr>
<td>/Rozier/formulas</td>
<td>364.475</td>
<td>50066.122</td>
<td>3918.415</td>
<td>6663.472</td>
<td>463.271</td>
<td>343.32</td>
</tr>
<tr>
<td>/Rozier/pattern</td>
<td>15.13</td>
<td>5530.001</td>
<td>17644.459</td>
<td>31.484</td>
<td>28.592</td>
<td>34.332</td>
</tr>
<tr>
<td>/schuppan/O1formula</td>
<td>1026.916</td>
<td>1148.885</td>
<td>12.681</td>
<td>6546.099</td>
<td>1356.775</td>
<td>882.993</td>
</tr>
<tr>
<td>/schuppan/O2formula</td>
<td>1082.35</td>
<td>1591.756</td>
<td>2167.806</td>
<td>1622.142</td>
<td>6.447</td>
<td>7.359</td>
</tr>
<tr>
<td>/schuppan/pftl</td>
<td>900.997</td>
<td>1810.009</td>
<td>1555.264</td>
<td>1081.194</td>
<td>3771.958</td>
<td>725.139</td>
</tr>
<tr>
<td>/trp/N5x</td>
<td>14.44</td>
<td>12575.152</td>
<td>12.681</td>
<td>6546.099</td>
<td>1356.775</td>
<td>31.627</td>
</tr>
<tr>
<td>/trp/N5y</td>
<td>2761.521</td>
<td>8983.292</td>
<td>1295.545</td>
<td>2763.737</td>
<td>2766.555</td>
<td>1384.437</td>
</tr>
<tr>
<td>/trp/N12x</td>
<td>20572.63</td>
<td>34455.41</td>
<td>25878.431</td>
<td>10513.257</td>
<td>2319.798</td>
<td>2387.982</td>
</tr>
<tr>
<td>/trp/N12y</td>
<td>3127.099</td>
<td>22231.442</td>
<td>22807.655</td>
<td>4026.722</td>
<td>4033.153</td>
<td>4042.285</td>
</tr>
<tr>
<td>Total</td>
<td>44506.513</td>
<td>169667.268</td>
<td>112307.805</td>
<td>44250.091</td>
<td>30459.832</td>
<td>15332.828</td>
</tr>
</tbody>
</table>

\(^5\) http://www.rcsg.rice.edu/sharecore/sugar/

\(^6\) http://patterns.projects.cis.ksu.edu/documentation/patterns/ltl.shtml
seventh ones list the total execution time for the corresponding type of formulas. Theoretically speaking, Polsat should be always the best. But as seen from the table, there may be some deviations between the results from Polsat and those best from integrated tools. This is due to the overhead we have to pay on pre-processing the input formula for each integrated tool (different tools have different input formats). In the table we also highlight the benchmarks for which Polsat is faster than the best of all solvers. The reason is that individual solver may not be superior to all cases in one type formulas while Polsat gets the best from different solvers in the same type, which leads to better overall performance for some benchmarks.

To show the power of Polsat on hard problems, we present the experimental results on two type of formulas. First, we extend Rozier’s random formulas [RV10] in Schuppen-collected benchmark via enlarging the size of generated formulas, and choosing 500 cases for each size (the formula length from 100 to 200). Second, we test the random conjunction formulas varying on the number of conjunctions (1-20) and select 500 random cases for each conjunction. The experimental results are shown in Fig. 3 and Fig. 4. In the figures we use the cactus plot to show the relationship between the number of instances solved by tools (x-axis) and their total checking costs (y-axis, with the second unit). One can see clearly from the figures that Polsat solves more cases with the same time, and has the best overall performance for these benchmarks.

As the integrated tool of off-the-shelf solvers, Polsat also provides a platform for competitions of LTL satisfiability solvers. By observing the best result among different solvers, Polsat knows which solver performs best for a given type of formulas. For example, for the /alaska/lift formulas, the NuSMV-BMC
performs best; Aalta does the best job for /schuppan/O1formula and /schuppan/O2formula formulas. The other benefit of Polsat is to make integrated tools potentially to optimize their performances by utilizing the experimental results.

5 Conclusion

We present a portfolio LTL satisfiability checker as well as an LTL testing platform, Polsat, by integrating existing off-the-shelf LTL satisfiability solvers. The goal is to provide a best LTL satisfiability solver by fully exploiting the distributed/multicore systems. Our empirical experimental results show that Polsat can have good overall performance for many benchmarks.

References


A A Simple Demonstration

In this section we show how to use Polsat by a simple demonstration. We will explain the input and output information of Polsat as well as the parameters the tool provides.

Polsat is run on Linux or Unix operating systems. After successfully installed Polsat and all its integrated solvers, one can directly type "./polsat" in the shell command line. By default the following information will show up:

```
please input the formula:
```

This means Polsat is waiting for the input. After you type the formula, such as “a U b” in the shell, then Polsat will produce the following output information:

```
sat
from pltl
eclipse time: 0.001s
```

The first line tells that the input formula is satisfiable; The second line shows this result is from pltl solver; The third line displays the checking time is 0.001 seconds.

Alternatively, for the same case, one can directly type "./polsat "a U b"" in the command line, and will get the same result.

A.1 Evidence for the Satisfiable formula

Similar to most of existed LTL satisfiability solvers, Polsat provides an interface to show an “evidence” for the satisfiable formula. By using the same formula “a U b”, if one uses the parameter “-e” of Polsat, that means, type "./polsat -e "a U b"” in the command line, then the output becomes:

```
sat
(b)
from Aalta
eclipse time: 0.002s
```

Here “(b)” in the second line represents the infinite trace $b^\omega$: obviously $b^\omega \models aU b$ holds. When the input formula is unsatisfiable, the flag “-e” will be ignored. Note here that not all integrated solvers provide the evidences for satisfiable formulas, so Polsat is designed to get the evidences from the Aalta solver since this solver has the functionality.
B Examples

The motivation of Polsat comes from that, none of existed LTL satisfiability solvers perform best for all benchmarks. In other words, each solver has its own advantages on some kind of formulas. The implementation of Polsat confirms that it inherits all advantages of integrated solvers. In the following we show two cases. Since small formulas do not make large derivations among solvers, we choose the formulas of large size as the demonstration.

B.1 NuSMV-BMC performs best on lift formulas

The lift formulas is one benchmark from Schuppan-collected for the lift specification. The following lists one formula for the lift for three floors:

\[
\begin{align*}
G((f0 \rightarrow (!f1 \land !f2)) \land (f1 \rightarrow !f2)) & \\
!u & \land f0 & !b0 & !b1 & !b2 & !up & \\
G(u \leftrightarrow !u) & \\
G((u \rightarrow (f0 \leftrightarrow X(f0)) \land (f1 \leftrightarrow X(f1)) \land (f2 \leftrightarrow X(f2))) & \\
(f0 \rightarrow X((f0 \land f1))) & \land (f1 \rightarrow X((f0 \land f1))) & \\
(f2 \rightarrow X((f1 \land f2))) & \\
G(!u \rightarrow (b0 \leftrightarrow X(b0)) \land (b1 \leftrightarrow X(b1)) \land (b2 \leftrightarrow X(b2))) & \\
(b0 \land !f0) & \rightarrow X(b0) & \land (b1 \land !f1) & \rightarrow X(b1) & \\
(b2 \land !f2) & \rightarrow X(b2)) & \\
G(((f0 \land X(f0)) \rightarrow (up \leftrightarrow X(up))) & \\
((f1 \land X(f1)) \rightarrow (up \leftrightarrow X(up))) & \\
((f2 \land X(f2)) \rightarrow (up \leftrightarrow X(up))) & \land ((f0 \land X(f1)) \rightarrow up) & \\
((f1 \land X(f2)) \rightarrow up) & \land ((f1 \land X(f0)) \rightarrow !up) & \\
((f2 \land X(f1)) \rightarrow !up)) & \\
G((sb \leftrightarrow (b0 \lor b1 \lor b2)) & \\
G(((f0 \land !sb) \rightarrow (f0 \lor (sb \lor (F(f0) \land !up)))))) & \\
((f1 \land !sb) \rightarrow (f1 \lor (sb \lor (F(f0) \land !up)))) & \\
((f2 \land !sb) \rightarrow (f2 \lor (sb \lor (F(f0) \land !up)))) & \\
G((b0 \rightarrow F(f0)) \land (b1 \rightarrow F(f1)) \land (b2 \rightarrow F(f2))) &
\end{align*}
\]

Taking this formula as input, Polsat gives the following output:

sat
from NuSMV-BMC
eclipse time: 0.005s

Generally speaking, the SAT-based checking shows the best performance for lift formulas, since the bounded model checking technique is suitable for solving satisfiable formulas. The experiments also confirm that NuSMV-BMC performs almost best for satisfiable formulas.
B.2 Aalta performs best on /schuppan/O1formula formulas

Let us take another example on unsatisfiable formulas. The benchmark “/schuppan/O1formula” formulas are such representatives. The following shows a formula from this benchmark with the length of 100.

```
(((a1) | (b1)) & ((a2) | (b2)) & ((a3) | (b3)) &
((a4) | (b4)) & ((a5) | (b5)) & ((a6) | (b6)) &
((a7) | (b7)) & ((a8) | (b8)) & ((a9) | (b9)) &
((a10) | (b10)) & ((a11) | (b11)) & ((a12) | (b12)) &
((a13) | (b13)) & ((a14) | (b14)) & ((a15) | (b15)) &
((a16) | (b16)) & ((a17) | (b17)) & ((a18) | (b18)) &
((a19) | (b19)) & ((a20) | (b20)) & ((a21) | (b21)) &
((a22) | (b22)) & ((a23) | (b23)) & ((a24) | (b24)) &
((a25) | (b25)) & ((a26) | (b26)) & ((a27) | (b27)) &
((a28) | (b28)) & ((a29) | (b29)) & ((a30) | (b30)) &
((a31) | (b31)) & ((a32) | (b32)) & ((a33) | (b33)) &
((a34) | (b34)) & ((a35) | (b35)) & ((a36) | (b36)) &
((a37) | (b37)) & ((a38) | (b38)) & ((a39) | (b39)) &
((a40) | (b40)) & ((a41) | (b41)) & ((a42) | (b42)) &
((a43) | (b43)) & ((a44) | (b44)) & ((a45) | (b45)) &
((a46) | (b46)) & ((a47) | (b47)) & ((a48) | (b48)) &
((a49) | (b49)) & ((a50) | (b50)) & ((a51) | (b51)) &
((a52) | (b52)) & ((a53) | (b53)) & ((a54) | (b54)) &
((a55) | (b55)) & ((a56) | (b56)) & ((a57) | (b57)) &
((a58) | (b58)) & ((a59) | (b59)) & ((a60) | (b60)) &
((a61) | (b61)) & ((a62) | (b62)) & ((a63) | (b63)) &
((a64) | (b64)) & ((a65) | (b65)) & ((a66) | (b66)) &
((a67) | (b67)) & ((a68) | (b68)) & ((a69) | (b69)) &
((a70) | (b70)) & ((a71) | (b71)) & ((a72) | (b72)) &
((a73) | (b73)) & ((a74) | (b74)) & ((a75) | (b75)) &
((a76) | (b76)) & ((a77) | (b77)) & ((a78) | (b78)) &
((a79) | (b79)) & ((a80) | (b80)) & ((a81) | (b81)) &
((a82) | (b82)) & ((a83) | (b83)) & ((a84) | (b84)) &
((a85) | (b85)) & ((a86) | (b86)) & ((a87) | (b87)) &
((a88) | (b88)) & ((a89) | (b89)) & ((a90) | (b90)) &
((a91) | (b91)) & ((a92) | (b92)) & ((a93) | (b93)) &
((a94) | (b94)) & ((a95) | (b95)) & ((a96) | (b96)) &
((a97) | (b97)) & ((a98) | (b98)) & ((a99) | (b99)) &
((a100) | (b100)) & ((G c) & (X ! c)))
```

The output of Polsat is below:

```
unsat
from Aalta
eclipse time: 0.04s
```
This formula is unsatisfiable, and one can see it is the last term of the formula, $((G \, c) \& (X \, ! \, c))$, that makes the formula unsatisfiable. If a solver provides some heuristic strategies for unsatisfiable formulas, it can give the answer very quickly. Since Aalta integrates some novel strategies to boost the search efficiency, it performs best in this case.

C The Testing Integration Platform

Polsat is not only a portfolio LTL satisfiability solver, but also considered as a testing integration platform for the existed or new LTL satisfiability solvers. That is to say, given the input formula, Polsat allows all integrated solvers to run separately, and every solver will not be terminated until it finishes checking. The Polsat then outputs all results and eclipse time for the solvers. For example, by adding the parameter “-s” and taking the following formula as the input,

\[
a \& G((a \rightarrow (X(!((a)) \& X(X(a)))))) \& !(b) \& X(!((b)) \& G(((a \& !(b)) \rightarrow (X(X(b))) \& X(((!(a)) \& (b \rightarrow X(X(b)))) \& !(b) \rightarrow X(X(!((b)))) U a)))))) \& G(((a \& b) \rightarrow (X(X((!(b)))) \& X(((!(a)) \& (b \rightarrow X(X(b)))) \& !(b) \rightarrow X(X((!(b)))) U a))))))
\]

which is a counter formula from the benchmark /rozier in schuppan-collected, Polsat gives the output:

<table>
<thead>
<tr>
<th>Solver</th>
<th>Result</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>pltl</td>
<td>sat</td>
<td>0.001s</td>
</tr>
<tr>
<td>NuSMV-BMC</td>
<td>sat</td>
<td>0.0026s</td>
</tr>
<tr>
<td>NuSMV-BDD</td>
<td>sat</td>
<td>0.014s</td>
</tr>
<tr>
<td>TRP++</td>
<td>sat</td>
<td>0.034s</td>
</tr>
<tr>
<td>Aalta</td>
<td>sat</td>
<td>0.57s</td>
</tr>
</tbody>
</table>

In each line of the output, it shows respectively the checking result (sat or unsat) and the eclipse time for all solvers. With the above information, one can check whether the checking results are consistent from all solvers, and the executing gap among different solvers. Moreover, based on the concrete results, the tool developer may try to explore the reason of inefficiency of the tool for some benchmark, and thus optimize the tool further.

C.1 Formulas in A File

As a testing platform, another key functionality that Polsat supports is to allow to input a set of formulas stored in a file and to provide the statistics by running the integrated solvers separately. By adding the parameter “-sm file” to Polsat, it will read all formulas in the specified file as the inputs and run them separately. The final output of Polsat in this situation will be stored into an output file including the checking result and time for each formula. For example, when taken a set of 100 random formulas as inputs, Polsat gives the following output:
The generated file is output.txt.

C.2 Adding External Solvers

Polsat is designed to be an open platform such that it allows to import external LTL satisfiability solvers as well. It can be achieved by using the “-add solver-path” parameter of Polsat. For example, the solver ALASKA is not integrated in Polsat currently, so we can use the flag to import it: type “./polsat -add "../alaska/alaska"” in the shell command line, then the following information will show up:

..../alaska/alaska is added.
please input the formula:

Then the solver ALASKA is successfully added to Polsat.