

# Formal Developments of Examples in the Paper: “Formalizing Hybrid Systems with Event-B and the Rodin Platform”

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# Contents

<b>1</b>	<b>The Saw</b>	<b>1</b>
1.1	Machine . . . . .	1
1.1.1	m0 . . . . .	1
1.1.2	m1 . . . . .	2
1.1.3	m1.alternative . . . . .	3
<b>2</b>	<b>Nuclear Plant Cooling</b>	<b>5</b>
2.1	Context . . . . .	5
2.1.1	C.c0 . . . . .	5
2.2	Machine . . . . .	6
2.2.1	C.m0 . . . . .	6
2.2.2	C.m1 . . . . .	7
<b>3</b>	<b>Controlling Train</b>	<b>10</b>
3.1	One Train Model . . . . .	10
3.1.1	c0 . . . . .	10
3.1.2	m0 . . . . .	11
3.2	Multiple Train Model . . . . .	13
3.2.1	c0 . . . . .	13
3.2.2	c1 . . . . .	13
3.2.3	m0 . . . . .	14
3.2.4	m1 . . . . .	16
<b>4</b>	<b>Aircraft Collision Avoidance</b>	<b>20</b>
4.1	Context . . . . .	20
4.1.1	c0 . . . . .	20
4.1.2	c1 . . . . .	21
4.2	Machine . . . . .	22
4.2.1	m0 . . . . .	22
4.2.2	m1 . . . . .	23
<b>5</b>	<b>Water Tank</b>	<b>26</b>
5.1	Context . . . . .	26
5.1.1	c0 . . . . .	26
5.2	Machine . . . . .	27
5.2.1	m0 . . . . .	27
5.2.2	m1 . . . . .	31
5.2.3	m2 . . . . .	35

# Chapter 1

## The Saw

### 1.1 Machine

#### 1.1.1 m0

An Event-B Specification of m0  
Creation Date: 1Nov2013 @ 04:45:41 PM

**MACHINE** m0

**VARIABLES**

*x*

*up*

**INVARIANTS**

*inv0\_1* :  $x \in \{0, 1\}$

*inv0\_2* :  $up \in \text{BOOL}$

**EVENTS**

**Initialisation**

**begin**

*act1* :  $x := 0$

*act2* :  $up := \text{FALSE}$

**end**

**Event** *UP*  $\hat{=}$

**when**

*grd1* :  $x = 0$

*grd2* :  $up = \text{FALSE}$

**then**

*act1* :  $x := 1$

*act2* :  $up := \text{TRUE}$

**end**

**Event** *DN*  $\hat{=}$

**when**

*grd1* :  $x = 1$

```

    then   grd2 : up = TRUE
    act1 : x := 0
    act2 : up := FALSE
  end
END

```

### 1.1.2 m1

**An Event-B Specification of m1**  
**Creation Date: 1Nov2013 @ 04:45:41 PM**

**MACHINE** m1

**REFINES** m0

**VARIABLES**

*x\_c*

*now*

*b*

*up*

**INVARIANTS**

*inv1\_1* :  $x_c \in \mathbb{N} \leftrightarrow \mathbb{Z}$

*inv1\_2* :  $now \in dom(x_c)$

*inv1\_3* :  $b = FALSE \Rightarrow x_c(now) = x$

*inv1\_11* :  $b = TRUE \Rightarrow now + 1 \in dom(x_c)$

*inv1\_4* :  $b = TRUE \Rightarrow x_c(now + 1) = x$

*inv1\_5* :  $b = TRUE \wedge up = TRUE \Rightarrow (x_c = \lambda t \cdot t \in \mathbb{N} \wedge t \geq now \wedge t \leq now + 1 | t - now)$

*inv1\_6* :  $b = TRUE \wedge up = FALSE \Rightarrow (x_c = \lambda t \cdot t \in \mathbb{N} \wedge t \geq now \wedge t \leq now + 1 | 1 - t + now)$

*inv1\_7* :  $b = FALSE \wedge up = TRUE \Rightarrow (x_c = \lambda t \cdot t \in \mathbb{N} \wedge t \geq now - 1 \wedge t \leq now | t - now + 1)$

*inv1\_8* :  $b = FALSE \wedge up = FALSE \Rightarrow (x_c = \lambda t \cdot t \in \mathbb{N} \wedge t \geq now - 1 \wedge t \leq now | -t + now)$

*thm1\_1* :  $b = TRUE$

$\Rightarrow$

$min(\{t | t \in dom(x_c) \wedge t \geq now \wedge ((x_c(t) = 0 \wedge up = FALSE) \vee (x_c(t) = 1 \wedge up = TRUE))\}) = now + 1$

*inv1\_9* :  $ran(x_c) \subseteq \{0, 1\}$

*inv1\_10* :  $b = TRUE \Rightarrow \neg((x_c(now) = 0 \wedge up = FALSE) \vee (x_c(now) = 1 \wedge up = TRUE))$

**EVENTS**

**Initialisation**

**begin**

*act3* :  $x_c := \{0 \mapsto 0\}$

*act2* :  $now := 0$

*act5* :  $up := FALSE$

*act4* :  $b := FALSE$

**end**

```
Event UP  $\hat{=}$ 
refines UP
  when
    grd1 :  $x\_c(now) = 0$ 
    grd3 :  $up = FALSE$ 
    grd2 :  $b = FALSE$ 
  then
    act1 :  $x\_c := \lambda t \cdot t \in \mathbb{N} \wedge t \geq now \wedge t \leq now + 1 | t - now$ 
    act3 :  $up := TRUE$ 
    act2 :  $b := TRUE$ 
  end
Event DN  $\hat{=}$ 
refines DN
  when
    grd1 :  $x\_c(now) = 1$ 
    grd3 :  $up = TRUE$ 
    grd2 :  $b = FALSE$ 
  then
    act1 :  $x\_c := \lambda t \cdot t \in \mathbb{N} \wedge t \geq now \wedge t \leq now + 1 | 1 - t + now$ 
    act3 :  $up := FALSE$ 
    act2 :  $b := TRUE$ 
  end
Event click  $\hat{=}$ 
  when
    grd1 :  $b = TRUE$ 
  then
    act1 :  $b := FALSE$ 
    act2 :  $now := now + 1$ 
  end
END
```

### 1.1.3 m1\_alternative

**An Event-B Specification of m1\_alternative**  
**Creation Date: 1Nov2013 @ 04:45:41 PM**

**MACHINE** m1\_alternative

**REFINES** m0

**VARIABLES**

*x\_c*

*now*

*up*

**INVARIANTS**

*inv2\_1* :  $x\_c \in \mathbb{N} \rightarrow \mathbb{Z}$

$inv2\_0 : now \in \mathbb{N}$   
 $inv2\_2 : now \in dom(x\_c)$   
 $inv2\_3 : x = x\_c(now)$   
 $inv2\_4 : up = TRUE \Rightarrow (x\_c = \lambda t.t \in \mathbb{N} \wedge t \geq now - 1 \wedge t \leq now | t - now + 1)$   
 $inv2\_5 : up = FALSE \Rightarrow (x\_c = \lambda t.t \in \mathbb{N} \wedge t \geq now - 1 \wedge t \leq now | -t + now)$   
 $thm2\_1 : min(\{t | t \geq now \wedge ((1 - t + now = 0 \wedge up = FALSE) \vee (t - now = 1 \wedge up = TRUE))\}) = now + 1$

**EVENTS**

**Initialisation**

**begin**

$act1 : now := 0$   
 $act2 : x\_c := \{0 \mapsto 0\}$   
 $act3 : up := FALSE$

**end**

**Event**  $UP \hat{=}$

**refines**  $UP$

**when**

$grd1 : x\_c(now) = 0$   
 $grd2 : up = FALSE$

**then**

$act1 : x\_c := \lambda t.t \in \mathbb{N} \wedge t \geq now \wedge t \leq now + 1 | t - now$   
 $act3 : up := TRUE$   
 $act2 : now := now + 1$

**end**

**Event**  $DN \hat{=}$

**refines**  $DN$

**when**

$grd1 : x\_c(now) = 1$   
 $grd2 : up = TRUE$

**then**

$act1 : x\_c := \lambda t.t \in \mathbb{N} \wedge t \geq now \wedge t \leq now + 1 | 1 - t + now$   
 $act3 : up := FALSE$   
 $act2 : now := now + 1$

**end**

**END**

## Chapter 2

# Nuclear Plant Cooling

### 2.1 Context

#### 2.1.1 C\_c0

An Event-B Specification of C\_c0  
Creation Date: 1Nov2013 @ 04:45:41 PM

**CONTEXT** C\_c0

**CONSTANTS**

*theta\_m*

*theta\_M*

*v1*

*v2*

*vr*

*T*

*a*

*b1*

*b2*

**AXIOMS**

*axm0\_1* :  $theta\_m \in \mathbb{N}$

*axm0\_2* :  $theta\_M \in \mathbb{N}$

*axm0\_3* :  $theta\_M > theta\_m$

*axm0\_4* :  $v1 > 0$

*axm0\_5* :  $v2 > 0$

*axm0\_6* :  $vr > 0$

*axm0\_7* :  $T > 0$

*axm1* :  $a > 0$

*axm2* :  $b1 > 0$

*axm3* :  $b2 > 0$

*axm0\_8* :  $vr * a = theta\_M - theta\_m$

*axm0\_9* :  $v1 * b1 = theta\_M - theta\_m$

$axm0_{10} : v2 * b2 = theta\_M - theta\_m$

$axm0_{11} : a < T$

$axm0_{12} : 2 * a + b1 \geq T$

$axm0_{13} : 2 * a + b2 \geq T$

**END**

## 2.2 Machine

### 2.2.1 C\_m0

**An Event-B Specification of C\_m0**  
**Creation Date: 1Nov2013 @ 04:45:41 PM**

**MACHINE** C\_m0

**SEES** C\_c0

**VARIABLES**

*theta*

*t1*

*t2*

*rod*

**INVARIANTS**

$inv0_{1} : theta \in \mathbb{N}$

$inv0_{2} : t1 \in \mathbb{N}$

$inv0_{3} : t2 \in \mathbb{N}$

$inv0_{4} : rod \in \{0, 1, 2\}$

$inv0_{5} : theta = theta\_M \Rightarrow t1 \geq T \vee t2 \geq T$

$inv0_{6} : rod = 1 \Rightarrow t2 = a + b1$

$inv0_{7} : rod = 2 \Rightarrow t1 = a + b2$

$inv0_{8} : rod = 0 \Rightarrow t1 + a \geq T \vee t2 + a \geq T$

$inv6 : rod = 1 \Rightarrow t2 + a \geq T$

$inv7 : rod = 2 \Rightarrow t1 + a \geq T$

$inv8 : rod = 0 \wedge t1 \geq T \Rightarrow t2 = a$

$inv9 : rod = 0 \wedge t2 \geq T \Rightarrow t1 = a$

$inv15 : theta = theta\_M \Rightarrow rod = 0$

Needed to prove inv10

Needed to prove inv10

Needed to prove inv8

Needed to prove inv9

**EVENTS**

**Initialisation**

**begin**

$act1 : t1 := 2 * a + b2$

$act2 : t2 := a$

$act4 : theta := theta\_M$

$act5 : rod := 0$

**end**

**Event** *cool\_rod1*  $\hat{=}$



```
    when
      grd2 :  $t1 \geq T$ 
      grd3 :  $theta = theta\_M$ 
    then
      act1 :  $rod := 1$ 
      act3 :  $t2 := t2 + b1$ 
      act4 :  $theta := theta\_m$ 
    end
  Event release_rod1  $\hat{=}$ 
    when
      grd1 :  $rod = 1$ 
      grd2 :  $theta = theta\_m$ 
    then
      act1 :  $rod := 0$ 
      act2 :  $t1 := a$ 
      act3 :  $t2 := t2 + a$ 
      act4 :  $theta := theta\_M$ 
    end
  Event cool_rod2  $\hat{=}$ 
    when
      grd2 :  $t2 \geq T$ 
      grd3 :  $theta = theta\_M$ 
    then
      act1 :  $rod := 2$ 
      act3 :  $t1 := t1 + b2$ 
      act4 :  $theta := theta\_m$ 
    end
  Event release_rod2  $\hat{=}$ 
    when
      grd1 :  $rod = 2$ 
      grd2 :  $theta = theta\_m$ 
    then
      act1 :  $rod := 0$ 
      act2 :  $t1 := t1 + a$ 
      act3 :  $t2 := a$ 
      act4 :  $theta := theta\_M$ 
    end
  END
```

### 2.2.2 C\_m1

An Event-B Specification of C_m1 Creation Date: 1Nov2013 @ 04:45:41 PM
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**MACHINE** C\_m1

**REFINES** C\_m0

**SEES** C\_c0

**VARIABLES**

*t1\_c*

*t2\_c*

*theta\_c*

*now*

*rod*

**INVARIANTS**

*inv1\_1* :  $t1\_c \in \mathbb{N} \leftrightarrow \mathbb{Z}$

*inv1\_2* :  $t2\_c \in \mathbb{N} \leftrightarrow \mathbb{Z}$

*inv1\_3* :  $theta\_c \in \mathbb{N} \leftrightarrow \mathbb{Z}$

*inv1\_4* :  $now \in dom(t1\_c) \cap dom(t2\_c) \cap dom(theta\_c)$

*inv1\_5* :  $t1 = t1\_c(now)$

*inv1\_6* :  $t2 = t2\_c(now)$

*inv1\_7* :  $theta = theta\_c(now)$

*thm1\_1* :  $rod = 1 \Rightarrow \min(\{t | t \geq now \wedge t = now + b1\}) = now + b1$

*thm1\_2* :  $rod = 2 \Rightarrow \min(\{t | t \geq now \wedge t = now + b2\}) = now + b2$

*thm1\_3* :  $rod = 0 \Rightarrow t1\_c(now) + a \geq T \vee t2\_c(now) + a \geq T$

*thm1\_4* :  $rod = 0 \Rightarrow \min(\{t | t \geq now \wedge (t1\_c(now) + t - now \geq T \vee t2\_c(now) + t - now \geq T) \wedge (t = now + a)\}) = now + a$

**EVENTS**

**Initialisation**

**begin**

*act1* :  $t1\_c := \{0 \mapsto 2 * a + b2\}$

*act2* :  $t2\_c := \{0 \mapsto a\}$

*act3* :  $rod := 0$

*act5* :  $now := 0$

*act6* :  $theta\_c := \{0 \mapsto theta\_M\}$

**end**

**Event** *cool\_rod1*  $\hat{=}$

**refines** *cool\_rod1*

**when**

*grd2* :  $t1\_c(now) \geq T$

*grd3* :  $theta\_c(now) = theta\_M$

*grd4* :  $b1 \in \mathbb{N}$

**then**

*act1* :  $rod := 1$

*act3* :  $t2\_c := \lambda t \cdot t \in now .. now + b1 | t2\_c(now) + (t - now)$

*act5* :  $t1\_c := \lambda t \cdot t \in now .. now + b1 | t1\_c(now)$

*act6* :  $theta\_c := \lambda t \cdot t \in now .. now + b1 | theta\_M - v1 * (t - now)$

*act4* :  $now := now + b1$

```
end
Event release_rod1  $\hat{=}$ 
refines release_rod1
  when
    grd1 : rod = 1
    grd2 : theta_c(now) = theta_m
    grd3 : a  $\in$   $\mathbb{N}$ 
  then
    act1 : rod := 0
    act2 : t1_c :=  $\lambda t \cdot t \in \text{now} .. \text{now} + a | (t - \text{now})$ 
    act3 : t2_c :=  $\lambda t \cdot t \in \text{now} .. \text{now} + a | t2\_c(\text{now}) + (t - \text{now})$ 
    act5 : theta_c :=  $\lambda t \cdot t \in \text{now} .. \text{now} + a | \text{theta}_m + vr * (t - \text{now})$ 
    act4 : now := now + a
  end
Event cool_rod2  $\hat{=}$ 
refines cool_rod2
  when
    grd2 : t2_c(now)  $\geq T$ 
    grd3 : theta_c(now) = theta_M
    grd4 : b2  $\in$   $\mathbb{N}$ 
  then
    act1 : rod := 2
    act3 : t1_c :=  $\lambda t \cdot t \in \text{now} .. \text{now} + b2 | t1\_c(\text{now}) + (t - \text{now})$ 
    act5 : t2_c :=  $\lambda t \cdot t \in \text{now} .. \text{now} + b2 | t2\_c(\text{now})$ 
    act6 : theta_c :=  $\lambda t \cdot t \in \text{now} .. \text{now} + b2 | \text{theta}_M - v2 * (t - \text{now})$ 
    act4 : now := now + b2
  end
Event release_rod2  $\hat{=}$ 
refines release_rod2
  when
    grd1 : rod = 2
    grd2 : theta_c(now) = theta_m
    grd3 : a  $\in$   $\mathbb{N}$ 
  then
    act1 : rod := 0
    act2 : t1_c :=  $\lambda t \cdot t \in \text{now} .. \text{now} + a | t1\_c(\text{now}) + (t - \text{now})$ 
    act3 : t2_c :=  $\lambda t \cdot t \in \text{now} .. \text{now} + a | (t - \text{now})$ 
    act5 : theta_c :=  $\lambda t \cdot t \in \text{now} .. \text{now} + a | \text{theta}_m + vr * (t - \text{now})$ 
    act4 : now := now + a
  end
end
END
```

# Chapter 3

## Controlling Train

### 3.1 One Train Model

#### 3.1.1 c0

An Event-B Specification of T\_c0  
Creation Date: 1Nov2013 @ 04:45:41 PM

**CONTEXT** T\_c0

**CONSTANTS**

*A*

*b*

*sl*

*e*

*m*

**AXIOMS**

*axm0\_1* :  $A > 0$

*axm0\_2* :  $b > 0$

*axm0\_3* :  $sl > 0$

*axm0\_4* :  $e > 0$

*axm0\_5* :  $m > 0$

*axm5* :  $\forall x, y. \neg y = 0 \Rightarrow y * (x/y) = x$

*axm6* :  $b = 1$

*axm7* :  $A = 2$

*axm8* :  $A * b = 2$

*axm9* :  $e = 2$

*axm11* :  $\forall x. x \in \mathbb{N} \Rightarrow 2 * (x * x/2) \in \mathbb{N}$

*axm12* :  $\forall x. x \in \mathbb{N} \Rightarrow x * x/2 \in \mathbb{N}$

**END**

## 3.1.2 m0

**An Event-B Specification of T\_m0**  
**Creation Date: 1Nov2013 @ 04:45:41 PM**

**MACHINE** T\_m0

One train behind goal

**SEES** T\_c0**VARIABLES** $z$  $v$  $a$  $phase$ **INVARIANTS** $inv0\_1 : z \in \mathbb{N}$  $inv0\_2 : v \in \mathbb{N}$  $inv0\_3 : a \in \{0, A, -b\}$  $inv0\_4 : phase \in \{1, 2\}$  $inv0\_5 : 2 * b * (m - z) \geq v * v$  $inv0\_6 : v \in 0 .. sl$  $inv6 : m - z \geq 0$ SAFETY CONDITION: the train position  $z$  is  
always situated BEFORE the limit position  $m$  $inv8 : phase = 2 \wedge v + a * e \leq 0 \Rightarrow a = -b$  $inv9 : phase = 2 \wedge a = A \Rightarrow 2 * b * (m - z) \geq v * v + (A * e * e + 2 * v * e) * (A + b)$  $inv11 : phase = 2 \wedge a = 0 \Rightarrow 2 * b * (m - z) \geq v * v + (A * e * e + 2 * v * e) * (A + b)$ **EVENTS****Initialisation****begin** $act1 : phase := 1$  $act3 : z := 0$  $act4 : v := 0$  $act5 : a := 2$ **end****Event**  $decide\_1 \hat{=}$ Decelerate with  $-b$ **when** $grd1 : phase = 1$  $grd2 : 2 * b * (m - z) < v * v + (A * e^2 + 2 * v * e) * (A + b)$  $grd4 : v > 0$ **then** $act1 : a := -b$  $act2 : phase := 2$ **end****Event**  $decide\_2 \hat{=}$ Accelerate with  $A$

```

when
    grd1 :  $phase = 1$ 
    grd2 :  $2 * b * (m - z) \geq v * v + (A * e * e + 2 * v * e) * (A + b)$ 
    grd3 :  $v + A * e \leq sl$ 
        Resulting speed smaller than or equal to sl
then
    act1 :  $a := A$ 
        a := A (A=2)
    act2 :  $phase := 2$ 
end
Event decide_3  $\hat{=}$ 
    Acceleration 0
when
    grd1 :  $phase = 1$ 
    grd2 :  $2 * b * (m - z) \geq v * v + (A * e * e + 2 * v * e) * (A + b)$ 
    grd3 :  $v + A * e > sl$ 
        Resulting speed greater than sl
    grd7 :  $v > 0$ 
then
    act1 :  $a := 0$ 
    act2 :  $phase := 2$ 
end
Event drive_1  $\hat{=}$ 
    Reaching v=0
when
    grd1 :  $phase = 2$ 
    grd2 :  $v + a * e \leq 0$ 
    grd3 :  $a = -b$ 
    grd4 :  $2 * (v * v / 2) = v * v$ 
then
    act1 :  $v := 0$ 
    act2 :  $z := z + (v * v) / 2$ 
    act3 :  $phase := 1$ 
end
Event drive_2  $\hat{=}$ 
    Proofs have to be done by cases (a=A, a=-b, or a=0)
when
    grd2 :  $phase = 2$ 
    grd3 :  $v + a * e > 0$ 
    grd4 :  $v \geq 0$ 
then
    act1 :  $v := v + a * e$ 
    act2 :  $z := z + v * e + a * e * e / 2$ 
    act3 :  $phase := 1$ 
end
END

```

## 3.2 Multiple Train Model

### 3.2.1 c0

An Event-B Specification of c0  
Creation Date: 1Nov2013 @ 04:45:41 PM

**CONTEXT** c0

**SETS**

*ID*

**CONSTANTS**

*trains*

*A*

*b*

*e*

*sl*

**AXIOMS**

axm1 :  $trains \subseteq \mathbb{N}$

axm2 :  $A > 0$

axm3 :  $b > 0$

axm6 :  $e > 0$

axm8 :  $sl > 0$

axm4 :  $b = 1$

axm5 :  $A = 2$

axm7 :  $e = 2$

axm9 :  $\forall x, y. \neg y = 0 \Rightarrow y * (x/y) = x$

axm10 :  $\forall x. x \in \mathbb{N} \Rightarrow 2 * (x * x/2) \in \mathbb{N}$

axm11 :  $\forall x. x \in \mathbb{N} \Rightarrow x * x/2 \in \mathbb{N}$

axm12 :  $\forall x, y. x \in \mathbb{Z} \wedge y \in \mathbb{Z} \wedge 2 * x \geq y * y \Rightarrow x \geq 0$

**END**

### 3.2.2 c1

An Event-B Specification of c1  
Creation Date: 1Nov2013 @ 04:45:41 PM

**CONTEXT** c1

**EXTENDS** c0

**CONSTANTS**

*c*

**AXIOMS**

axm2 :  $c > 0$

axm3 :  $trains = 0 .. c$

**END**

## 3.2.3 m0

An Event-B Specification of m0  
Creation Date: 1Nov2013 @ 04:45:41 PM

**MACHINE** m0**SEES** c0**VARIABLES**

*phase*  
*position*  
*acc*  
  
*speed*  
*limit*

**INVARIANTS**

*inv3* :  $position \in trains \rightarrow \mathbb{Z}$   
*inv5* :  $speed \in trains \rightarrow \mathbb{N}$   
*inv4* :  $acc \in trains \rightarrow \{0, A, -b\}$   
*inv2* :  $phase \in trains \rightarrow \{0, 1, 2\}$   
*inv6* :  $limit \in trains \rightarrow \mathbb{N}$   
*inv7* :  $\forall i \cdot i \in trains \Rightarrow 2 * b * (limit(i) - position(i)) \geq speed(i) * speed(i)$   
*inv12* :  $\forall i \cdot i \in trains \Rightarrow speed(i) \in 0 .. sl$   
*inv8* :  $\forall i \cdot i \in trains \Rightarrow limit(i) - position(i) \geq 0$   
*inv9* :  $\forall i \cdot i \in trains \Rightarrow (phase(i) = 2 \wedge speed(i) + 2 * acc(i) \leq 0 \Rightarrow acc(i) = -b)$   
*inv10* :  $\forall i \cdot i \in trains \Rightarrow (phase(i) = 2 \wedge acc(i) = A \Rightarrow 2 * (limit(i) - position(i)) \geq speed(i) * speed(i) + 24 + 12 * speed(i))$   
*inv11* :  $\forall i \cdot i \in trains \Rightarrow (phase(i) = 2 \wedge acc(i) = 0 \Rightarrow 2 * limit(i) - 2 * position(i) \geq speed(i) * speed(i) + 24 + 12 * speed(i))$   
*inv13* :  $\forall i \cdot i \in trains \Rightarrow (phase(i) = 2 \wedge acc(i) = A \Rightarrow speed(i) + 4 \leq sl)$   
*inv14* :  $\forall i \cdot i \in trains \Rightarrow (phase(i) = 2 \wedge speed(i) + e * acc(i) \leq 0 \Rightarrow acc(i) = -b)$   
*inv15* :  $\forall i \cdot i \in trains \Rightarrow (phase(i) = 2 \wedge acc(i) = A \Rightarrow 2 * (limit(i) - position(i)) \geq speed(i) * speed(i) + (A * e * e + 2 * e * speed(i)) * (A + b))$   
*inv16* :  $\forall i \cdot i \in trains \Rightarrow (phase(i) = 2 \wedge acc(i) = 0 \Rightarrow 2 * limit(i) - 2 * position(i) \geq speed(i) * speed(i) + (A * e * e + 2 * e * speed(i)) * (A + b))$

**EVENTS****Initialisation****begin**

*act2* :  $phase := trains \times \{1\}$   
*act3* :  $position := trains \times \{0\}$   
*act4* :  $acc := trains \times \{0\}$   
*act5* :  $speed := trains \times \{0\}$   
*act6* :  $limit := trains \times \{0\}$

**end****Event** *get\_limit*  $\hat{=}$



```

any
     $x$ 
where
     $i$ 
    grd1 :  $x \in 1..300$ 
    grd2 :  $i \in \text{trains}$ 
then
    act1 :  $\text{limit}(i) := \text{limit}(i) + x$ 
end
Event decide_1  $\hat{=}$ 
any
     $i$ 
where
    grd5 :  $i \in \text{trains}$ 
    grd1 :  $\text{phase}(i) = 1$ 
    grd2 :  $2 * \text{limit}(i) - 2 * \text{position}(i) < \text{speed}(i) * \text{speed}(i) + (A * e * e + 2 * e * \text{speed}(i)) * (A + b)$ 
    grd3 :  $A > 0$ 
    grd4 :  $\text{speed}(i) > 0$ 
then
    act1 :  $\text{acc}(i) := -b$ 
    act2 :  $\text{phase}(i) := 2$ 
end
Event decide_2  $\hat{=}$ 
any
     $i$ 
where
    grd7 :  $i \in \text{trains}$ 
    grd1 :  $\text{phase}(i) = 1$ 
    grd2 :  $2 * \text{limit}(i) - 2 * \text{position}(i) \geq \text{speed}(i) * \text{speed}(i) + (A * e * e + 2 * e * \text{speed}(i)) * (A + b)$ 
    grd3 :  $\text{speed}(i) \geq 0$ 
    grd4 :  $e > 0$ 
    grd5 :  $A > 0$ 
    grd6 :  $\text{speed}(i) + e * A \leq sl$ 
then
    act1 :  $\text{acc}(i) := A$ 
    act2 :  $\text{phase}(i) := 2$ 
end
Event decide_3  $\hat{=}$ 
any
     $i$ 
where
    grd1 :  $i \in \text{trains}$ 
    grd2 :  $\text{phase}(i) = 1$ 
    grd3 :  $2 * \text{limit}(i) - 2 * \text{position}(i) \geq \text{speed}(i) * \text{speed}(i) + (A * e * e + 2 * e * \text{speed}(i)) * (A + b)$ 

```

```

    grd4 :  $speed(i) > 0$ 
    grd5 :  $speed(i) + e * A > sl$ 
  then
    act1 :  $acc(i) := 0$ 
    act2 :  $phase(i) := 2$ 
  end
Event drive1  $\hat{=}$ 
  any
  where i
    grd1 :  $i \in trains$ 
    grd2 :  $phase(i) = 2$ 
    grd3 :  $speed(i) + e * acc(i) \leq 0$ 
           e=2
  then
    act1 :  $speed(i) := 0$ 
    act2 :  $position(i) := position(i) + (speed(i) * speed(i))/(2 * b)$ 
    act3 :  $phase(i) := 1$ 
  end
Event drive2  $\hat{=}$ 
  any
  where i
    grd1 :  $i \in trains$ 
    grd2 :  $phase(i) = 2$ 
    grd3 :  $speed(i) + e * acc(i) > 0$ 
           e=2
    grd4 :  $speed(i) \geq 0$ 
  then
    act1 :  $speed(i) := speed(i) + e * acc(i)$ 
    act2 :  $position(i) := position(i) + e * speed(i) + acc(i) * (e * e/2)$ 
    act3 :  $phase(i) := 1$ 
  end
end
END

```

### 3.2.4 m1

An Event-B Specification of m1  
 Creation Date: 1Nov2013 @ 04:45:41 PM

```

MACHINE m1
REFINES m0
SEES c1
VARIABLES

```

*phase*  
*position*  
*acc*  
*speed*  
*limit*

**INVARIANTS**

*inv3* :  $\forall j \cdot j \in 1 .. c \Rightarrow \text{position}(j - 1) \geq \text{limit}(j)$   
*inv1* :  $\forall j \cdot j \in 1 .. c \Rightarrow \text{position}(j - 1) \geq \text{position}(j)$

**EVENTS****Initialisation***extended***begin**

*act2* : *phase* := *trains* × {1}  
*act3* : *position* := *trains* × {0}  
*act4* : *acc* := *trains* × {0}  
*act5* : *speed* := *trains* × {0}  
*act6* : *limit* := *trains* × {0}

**end****Event** *get\_limit\_first*  $\hat{=}$ **extends** *get\_limit***any***x**i***where**

*grd1* :  $x \in 1 .. 300$   
*grd2* :  $i \in \text{trains}$   
 $\text{limit}(i) - \text{position}(i) < 12, \text{limit}(i) = 0$   
*grd8* :  $i = 0$   
*grd7* :  $\text{limit}(i) - \text{position}(i) < 12$

**then***act1* : *limit*(*i*) := *limit*(*i*) + *x***end****Event** *get\_limit\_following*  $\hat{=}$ **refines** *get\_limit***any***i***where**

*grd2* :  $i \in \text{trains}$   
*grd3* :  $\top$   
 $\text{limit}(i) - \text{position}(i) < 12$   
*grd4* :  $\top$   
 $\text{limit}(i) = 0$   
*grd5* :  $i \in 1 .. c$

---

```

    with   grd1 :  $\min(\{position(i-1) - limit(i), 300\}) \in 1..300$ 
  then
    x :     $x = \min(\{ position(i-1) - limit(i), 300 \} )$ 
  then
    act1 :  $limit(i) := limit(i) + \min(\{position(i-1) - limit(i), 300\})$ 
  end
Event decide_1  $\hat{=}$ 
extends decide_1
  any
  where
    i
    grd5 :  $i \in trains$ 
    grd1 :  $phase(i) = 1$ 
    grd2 :  $2 * limit(i) - 2 * position(i) < speed(i) * speed(i) + (A * e * e + 2 * e * speed(i)) * (A + b)$ 
    grd3 :  $A > 0$ 
    grd4 :  $speed(i) > 0$ 
  then
    act1 :  $acc(i) := -b$ 
    act2 :  $phase(i) := 2$ 
  end
Event decide_2  $\hat{=}$ 
extends decide_2
  any
  where
    i
    grd7 :  $i \in trains$ 
    grd1 :  $phase(i) = 1$ 
    grd2 :  $2 * limit(i) - 2 * position(i) \geq speed(i) * speed(i) + (A * e * e + 2 * e * speed(i)) * (A + b)$ 
    grd3 :  $speed(i) \geq 0$ 
    grd4 :  $e > 0$ 
    grd5 :  $A > 0$ 
    grd6 :  $speed(i) + e * A \leq sl$ 
            $e=2, A=2$ 
  then
    act1 :  $acc(i) := A$ 
    act2 :  $phase(i) := 2$ 
  end
Event decide_3  $\hat{=}$ 
extends decide_3
  any
  where
    i
    grd1 :  $i \in trains$ 

```

---

```

    grd2 :  $phase(i) = 1$ 
    grd3 :  $2 * limit(i) - 2 * position(i) \geq speed(i) * speed(i) + (A * e * e + 2 * e * speed(i)) * (A + b)$ 
    grd4 :  $speed(i) > 0$ 
    grd5 :  $speed(i) + e * A > sl$ 
  then
    act1 :  $acc(i) := 0$ 
    act2 :  $phase(i) := 2$ 
  end
Event drive1  $\hat{=}$ 
extends drive1
  any
    where
      i
      grd1 :  $i \in trains$ 
      grd2 :  $phase(i) = 2$ 
      grd3 :  $speed(i) + e * acc(i) \leq 0$ 
      e=2
    then
      act1 :  $speed(i) := 0$ 
      act2 :  $position(i) := position(i) + (speed(i) * speed(i)) / (2 * b)$ 
      act3 :  $phase(i) := 1$ 
    end
Event drive2  $\hat{=}$ 
extends drive2
  any
    where
      i
      grd1 :  $i \in trains$ 
      grd2 :  $phase(i) = 2$ 
      grd3 :  $speed(i) + e * acc(i) > 0$ 
      e=2
      grd4 :  $speed(i) \geq 0$ 
    then
      act1 :  $speed(i) := speed(i) + e * acc(i)$ 
      act2 :  $position(i) := position(i) + e * speed(i) + acc(i) * (e * e / 2)$ 
      act3 :  $phase(i) := 1$ 
    end
  end
END

```

## Chapter 4

# Aircraft Collision Avoidance

### 4.1 Context

#### 4.1.1 c0

An Event-B Specification of f.c0  
Creation Date: 1Nov2013 @ 04:45:41 PM

**CONTEXT** f\_c0

**CONSTANTS**

*rhoi*

*v*

*p*

*phi*

*r*

*sqrt*

*sin*

*phi\_d\_2*

**AXIOMS**

**axm0\_1** :  $rhoi > 0$

Initial distance of both aircrafts

**axm0\_2** :  $v > 0$

**axm0\_3** :  $p > 0$

minimal distance between aircrafts

**axm0\_4** :  $phi > 0$

**axm0\_6** :  $sqrt \in \mathbb{N} \rightarrow \mathbb{N}$

**axm0\_7** :  $sin \in \mathbb{Z} \rightarrow -1 .. 1$

**axm0\_5** :  $2 * rhoi * sin(phi_d_2) \geq p * sqrt(3)$

Initial distance greater than  $p * sqrt(3)$

**axm0\_8** :  $r > 0$

**axm0\_9** :  $p \leq 2 * r * sin(phi_d_2)$

$axm0_{-}10 : r * sqrt(3) \leq rhoi$   
 $axm8 : sqrt(3) > 1$   
 $axm9 : sqrt(3) * sqrt(3) = 3$   
 $axm15 : sin(phi\_d\_2) > 0$   
 $axm16 : phi\_d\_2 = phi/2$   
 $axm20 : 2 * rhoi * sin(phi\_d\_2) \geq p$

**END**

#### 4.1.2 c1

**An Event-B Specification of f.c1**  
**Creation Date: 1Nov2013 @ 04:45:41 PM**

**CONTEXT** f\_c1

**EXTENDS** f\_c0

**CONSTANTS**

$t1$   
 $t2$   
 $pi$   
 $t3$   
 $cos$

**AXIOMS**

$axm3 : pi \in 3 .. 4$   
 $axm1 : t1 = (rhoi - sqrt(3) * r) / v$   
 $axm2 : t2 = (pi * r) / (3 * v)$   
 $axm4 : t3 = (2 * pi * r) / (3 * v)$   
 $axm9 : t1 \geq 0$   
 $axm7 : t2 \geq 0$   
 $axm8 : t3 \geq 0$   
 $axm5 : cos \in \mathbb{Z} \rightarrow -1 .. 1$   
 $axm6 : v * t1 = rhoi - sqrt(3) * r$   
 $axm12 : 5 - 4 * cos(v * t2 / r) \in dom(sqrt)$   
 $axm13 : 5 - 4 * cos(pi/3 - v * t2 / r) \in dom(sqrt)$   
 $axm10 : sqrt(5 - 4 * cos(pi/3 - v * t2 / r)) = 1$   
 $axm11 : sqrt(5 - 4 * cos(v * t2 / r)) = sqrt(3)$   
 $axm14 : \forall t. t \in 0 .. t1 \Rightarrow 2 * (rhoi - v * t) * sin(phi\_d\_2) \geq p$   
 $axm17 : \forall t. t \in t1 .. t1 + t2$   
 $\Rightarrow$   
 $(5 - 4 * cos(pi/3 - v * (t - t1) / r)) \in dom(sqrt)$   
 $axm18 : \forall t. t \in t1 + t2 + t3 .. t1 + t2 + t3 + t2$   
 $\Rightarrow$   
 $5 - 4 * cos(v * (t - (t1 + t2 + t3)) / r) \in dom(sqrt)$

```

axm15 :  $\forall t \cdot t \in t1 .. t1 + t2$ 
         $\Rightarrow$ 
         $2 * r * \text{sqrt}(5 - 4 * \cos(\text{pi}/3 - v * (t - t1)/r)) * \sin(\text{phi}_d.2) \geq p$ 
axm16 :  $\forall t \cdot t \in t1 + t2 + t3 .. t1 + t2 + t3 + t2$ 
         $\Rightarrow$ 
         $2 * r * \text{sqrt}(5 - 4 * \cos(v * (t - (t1 + t2 + t3))/r)) * \sin(\text{phi}_d.2) \geq p$ 

```

END

## 4.2 Machine

### 4.2.1 m0

An Event-B Specification of f\_m0  
 Creation Date: 1Nov2013 @ 04:45:41 PM

**MACHINE** f\_m0

This is a simplified version.

We remove the event AGREE from the original model presented in the paper

**SEES** f\_c0

**VARIABLES**

*phase*

*rho*

**INVARIANTS**

*inv0\_1* :  $phase \in 1 .. 5$

*inv0\_2* :  $rho > 0$

*inv0\_4* :  $2 * rho * \sin(\text{phi}_d.2) \geq p$

Distance always greater than or equal to p

*inv1* :  $phase = 3 \Rightarrow rho = r$

**EVENTS**

**Initialisation**

**begin**

*act1* :  $rho := rho_i$

*act2* :  $phase := 1$

**end**

**Event** *start*  $\hat{=}$

**when**

*grd1* :  $phase = 1$

**then**

*act1* :  $phase := 2$

*act2* :  $rho := \text{sqrt}(3) * r$

**end**

**Event** *enter*  $\hat{=}$

**when**

*grd1* :  $phase = 2$



```

    then
        act1 : phase := 3
        act3 : rho := r
    end
Event cycle ≐
    when
        grd1 : phase = 3
    then
        act1 : phase := 4
    end
Event leave ≐
    when
        grd1 : phase = 4
    then
        act1 : phase := 5
        act3 : rho := r * sqrt(3)
    end
END

```

#### 4.2.2 m1

An Event-B Specification of f\_m1  
Creation Date: 1Nov2013 @ 04:45:41 PM

```

MACHINE f_m1
REFINES f_m0
SEES f_c1
VARIABLES

```

```

    rho_c
    phase
    now

```

#### INVARIANTS

```

inv1_1 : rho_c ∈ ℕ ↔ ℤ
inv1_2 : now ∈ dom(rho_c)
inv1_3 : rho = rho_c(now)
inv1_4 : ∀t. t ∈ dom(rho_c) ⇒ 2 * rho_c(t) * sin(phi_d.2) ≥ p
inv4 : phase = 1 ⇒ now = 0
inv7 : phase = 1 ⇒ rho_c = {0 ↦ rho_i}
inv9 : phase = 3 ⇒ dom(rho_c) = now - (pi * r)/(3 * v) .. now
inv11 : phase = 2 ⇒ now = t1
inv12 : phase = 3 ⇒ now = t1 + t2
inv13 : phase = 4 ⇒ now = t1 + t2 + t3

```

## EVENTS

### Initialisation

**begin**

*act5* : *phase* := 1  
*act8* : *now* := 0  
*act9* : *rho\_c* := {0 ↦ *rhoi*}

**end**

**Event** *start*  $\hat{=}$

**refines** *start*

**when**

**then** *grd1* : *phase* = 1

*act1* : *phase* := 2  
*act5* : *rho\_c* :=  $\lambda t \cdot t \in \text{now} .. \text{now} + t1 | \text{rhoi} - v * (t - \text{now})$   
*act6* : *now* := *now* + *t1*

**end**

**Event** *enter*  $\hat{=}$

**refines** *enter*

**when**

**then** *grd1* : *phase* = 2

*act1* : *phase* := 3  
*act7* : *now* := *now* + *t2*  
*act8* : *rho\_c* :=  $\lambda t \cdot t \in \text{now} .. \text{now} + t2 | r * \text{sqrt}(5 - 4 * \cos(\text{pi}/3 - (v * (t - \text{now})/r)))$

**end**

**Event** *cycle*  $\hat{=}$

**refines** *cycle*

**when**

**then** *grd1* : *phase* = 3  
*grd2* : *v* > 0  
*grd3* : *r* > 0  
*grd4* : *v* > 0

*act1* : *phase* := 4  
*act5* : *now* := *now* + *t3*  
*act6* : *rho\_c* :=  $\lambda t \cdot t \in \text{now} .. \text{now} + t3 | r$

**end**

**Event** *leave*  $\hat{=}$

**refines** *leave*

**when**

**then** *grd1* : *phase* = 4

*act1* : *phase* := 5

```
act7 : now := now + t2
act8 : rho_c :=  $\lambda t \cdot t \in now .. now + t2 | r * \text{sqrt}(5 - 4 * \cos(v * (t - now)/r))$ 
end
END
```

# Chapter 5

## Water Tank

### 5.1 Context

#### 5.1.1 c0

An Event-B Specification of w\_c0  
Creation Date: 1Nov2013 @ 04:45:41 PM

**CONTEXT** w\_c0

**SETS**

*P*

**CONSTANTS**

*on*

*off*

*LM*

*Lm*

*v1*

*v2*

*t\_sen*

*t\_act*

*Li*

*PUMPi*

**AXIOMS**

**axm1** :  $partition(P, \{on\}, \{off\})$

**axm2** :  $LM > 0$

**axm3** :  $Lm > 0$

**axm4** :  $v1 > 0$

**axm5** :  $v2 > 0$

**axm6** :  $t\_sen > 0$

**axm7** :  $t\_act > 0$

**axm8** :  $LM - v1 * t\_sen \geq Lm + v2 * t\_sen$

**axm9** :  $t\_sen > t\_act$

$axm10 : Lm < LM$   
 $thm1 : Lm + v2 * t_{act} < LM - v1 * t_{act}$   
 $thm2 : v1 * t_{act} - v2 * t_{sen} + v2 * t_{act} \leq v1 * t_{act}$   
 $thm3 : -v2 * t_{act} + v1 * t_{sen} - v1 * t_{act} \geq -v2 * t_{act}$   
 $axm11 : PUMPi = on \Rightarrow Lm \leq Li$   
 $axm16 : PUMPi = off \Rightarrow Li \leq LM$   
 $axm12 : PUMPi = on \Rightarrow Li \leq LM - v1 * t_{act}$   
 $axm13 : PUMPi = off \Rightarrow Lm + v2 * t_{act} \leq Li$   
 $axm14 : PUMPi = on \Rightarrow Li \in Lm .. LM$   
 $axm15 : PUMPi = off \Rightarrow Li \in Lm .. LM$

END

## 5.2 Machine

### 5.2.1 m0

An Event-B Specification of w\_m0  
 Creation Date: 1Nov2013 @ 04:45:41 PM

**MACHINE** w\_m0

In this abstraction, we suppose that the two mechanisms:

- (1) physical pump possible modification
- (2) sending of the water level

occur simultaneously.

In the next refinement, we shall separate these two mechanisms.

**SEES** w\_c0

**VARIABLES**

*PUMP*

*pump*

*L*

*phase*

**INVARIANTS**

$inv0\_1 : PUMP \in P$

$inv0\_2 : pump \in P$

$inv0\_3 : phase \in \{1, 2\}$

$inv0\_4 : phase = 1 \Rightarrow pump = PUMP$

When in the controller, the status of the physical PUMP agrees with the pump command that has been sent previously.

$inv0\_5 : L \in Lm .. LM$

The safety invariant

*inv0\_10* :  $PUMP = on \Rightarrow L \leq LM - v1 * t_{act}$

When the physical PUMP is working then the water level is always smaller than or equal to  $tM$  minus the quantity corresponding to the delay  $t_{act}$  (with speed  $v1$ ).

*inv0\_11* :  $PUMP = off \Rightarrow L \geq Lm + v2 * t_{act}$

When the physical PUMP is stopped then the water level is always greater than or equal to  $tM$  plus the quantity corresponding to the delay  $t_{act}$  (with speed  $v2$ ).

*inv0\_12* :

$phase = 2 \wedge$   
 $PUMP = on \wedge$   
 $pump = on$   
 $\Rightarrow$   
 $L + v1 * t_{sen} \leq LM - v1 * t_{act}$

*inv0\_13* :  $phase = 2 \wedge$

$PUMP = off \wedge$   
 $pump = off$   
 $\Rightarrow$   
 $L - v2 * t_{sen} \geq Lm + v2 * t_{act}$

*inv0\_14* :  $phase = 2 \wedge$

$PUMP = on \wedge$   
 $pump = off$   
 $\Rightarrow$   
 $Lm \leq L + v1 * t_{act} - v2 * t_{sen}$

*inv0\_15* :  $phase = 2 \wedge$

$PUMP = off \wedge$   
 $pump = on$   
 $\Rightarrow$   
 $L - v2 * t_{act} + v1 * t_{sen} \leq LM$

*thm0\_1* :  $L - v2 * t_{act} + v1 * t_{sen} - v1 * t_{act} < L - v2 * t_{act} + v1 * t_{sen}$

*thm0\_2* :  $L + v1 * t_{act} - v2 * t_{sen} < L + v1 * t_{act} - v2 * t_{sen} + v2 * t_{act}$

*thm0\_3* :  $phase = 2 \wedge$

$PUMP = on \wedge$   
 $pump = on$   
 $\Rightarrow$   
 $L + v1 * t_{sen} \leq LM$   
 ( *inv0\_6* in paper)

*thm0\_4* :  $phase = 2 \wedge$

$PUMP = off \wedge$   
 $pump = off$   
 $\Rightarrow$   
 $Lm \leq L - v2 * t_{sen}$   
 ( *inv0\_7* in paper)

$thm0\_5 : phase = 2 \wedge$   
 $PUMP = off \wedge$   
 $pump = on$   
 $\Rightarrow$   
 $L - v2 * t\_act + v1 * t\_sen - v1 * t\_act \leq LM$   
 ( inv0.8 in paper )  
 $thm0\_6 : phase = 2 \wedge$   
 $PUMP = on \wedge$   
 $pump = off$   
 $\Rightarrow$   
 $Lm \leq L + v1 * t\_act - v2 * t\_sen + v2 * t\_act$   
 ( inv0.9 in paper )

## EVENTS

### Initialisation

**begin**

$act1 : PUMP := PUMPi$   
 $act2 : pump := PUMPi$   
 $act3 : L := Li$   
 $act4 : phase := 1$

**end**

**Event**  $decide\_1 \hat{=}$

Do nothing. The pump should be still on.

**when**

$grd1 : phase = 1$   
 $grd2 : pump = on$   
 $grd3 : L + v1 * t\_sen \leq LM - v1 * t\_act$   
 The water level can continue to go up.  
 It will not be dangerous at next step in  
 $t\_sen$  seconds.

**then**

$act1 : phase := 2$

**end**

**Event**  $decide\_2 \hat{=}$

Send the command to stop the pump.

**when**

$grd1 : phase = 1$   
 $grd2 : pump = on$   
 $grd3 : L + v1 * t\_sen > LM - v1 * t\_act$   
 The water level cannot continue to go up.  
 It will be dangerous (too high) at next step  
 in  $t\_sen$  seconds.

$thm1 : L \leq LM - v1 * t\_act$

$thm2 : LM - v1 * t\_sen \geq Lm + v2 * t\_sen$

**then**

$act1 : pump := off$

$act2 : phase := 2$

**end**

**Event** *decide\_3*  $\hat{=}$

Do nothing. The pump should be still off.

**when**

*grd1* : *phase* = 1

*grd2* : *pump* = *off*

*grd3* :  $L - v2 * t\_sen \geq Lm + v2 * t\_act$

The water level can continue to go down.

It will not be dangerous at next step in

*t\_sen* seconds.

**then**

*act1* : *phase* := 2

**end**

**Event** *decide\_4*  $\hat{=}$

Send the command to start the pump.

**when**

*grd1* : *phase* = 1

*grd2* : *pump* = *off*

*grd3* :  $L - v2 * t\_sen < Lm + v2 * t\_act$

The water level cannot continue to go down.

It will be dangerous (too low) at next step

in *t\_sen* seconds.

*thm1* :  $L \geq Lm + v2 * t\_act$

*thm2* :  $LM - v1 * t\_sen \geq Lm + v2 * t\_sen$

**then**

*act1* : *pump* := *on*

*act2* : *phase* := 2

**end**

**Event** *env\_1*  $\hat{=}$

The pump state (on) is not changed. Send the water level

**when**

*grd1* : *phase* = 2

*grd2* : *PUMP* = *on*

*grd3* : *pump* = *on*

**then**

*act1* :  $L := L + v1 * t\_sen$

*act2* : *phase* := 1

**end**

**Event** *env\_2*  $\hat{=}$

The pump state (off) is not changed. Send the water level

**when**

*grd1* : *phase* = 2

*grd2* : *PUMP* = *off*

*grd3* : *pump* = *off*

**then**



```

    act1 :  $L := L - v2 * t\_sen$ 
    act2 :  $phase := 1$ 
  end
Event env_3  $\hat{=}$ 
    The pump is stopped. Send the water level
  when
    grd1 :  $phase = 2$ 
    grd2 :  $PUMP = on$ 
    grd3 :  $pump = off$ 
    The pump receives the command to stop.
    thm1 :  $L + v1 * t\_act \leq LM$ 
    thm2 :  $v1 * t\_act - v2 * t\_sen + v2 * t\_act \leq v1 * t\_act$ 
    thm3 :  $L + v1 * t\_act - v2 * t\_sen + v2 * t\_act \leq L + v1 * t\_act$ 
  then
    act1 :  $L := L + v1 * t\_act - v2 * t\_sen + v2 * t\_act$ 
    The water level continues to go up with
    speed v1 for a time t_act. Then it goes down
    with speed v2 for a time t_sen-t_act.
    act2 :  $phase := 1$ 
    act3 :  $PUMP := off$ 
  end
Event env_4  $\hat{=}$ 
    The pump is started. Send the water level.
  when
    grd1 :  $phase = 2$ 
    grd2 :  $PUMP = off$ 
    grd3 :  $pump = on$ 
    The pump receives the command to start.
    thm1 :  $Lm \leq L - v2 * t\_act$ 
    thm2 :  $-v2 * t\_act + v1 * t\_sen - v1 * t\_act \geq -v2 * t\_act$ 
    thm3 :  $L - v2 * t\_act \leq L - v2 * t\_act + v1 * t\_sen - v1 * t\_act$ 
    thm4 :  $Lm \leq L - v2 * t\_act + v1 * t\_sen - v1 * t\_act$ 
  then
    act2 :  $L := L - v2 * t\_act + v1 * t\_sen - v1 * t\_act$ 
    The water level continues to go down with
    speed v2 for a time t_act. Then it goes up
    with speed v1 for a time t_sen-t_act.
    act3 :  $phase := 1$ 
    act1 :  $PUMP := on$ 
  end
END

```

## 5.2.2 m1

An Event-B Specification of w\_m1  
 Creation Date: 1Nov2013 @ 04:45:41 PM

**MACHINE** w\_m1

In this refinement, we separate the modification of the physical pump and the sending of the water level

**REFINES** w\_m0

**SEES** w\_c0

**VARIABLES**

*pump*

*phase*

*LP*

*PUMPP*

*p\_done*

*change*

**INVARIANTS**

*inv1\_1* :  $LP \in Lm .. LM$

*inv1\_2* :  $PUMPP \in P$

*inv1\_3* :  $p\_done = TRUE \Rightarrow PUMPP = pump$

*inv1\_4* :  $phase = 1 \Rightarrow p\_done = FALSE$

*inv1\_5* :  $p\_done = FALSE \Rightarrow LP = L$

$phase=2 \wedge$

*inv1\_6* :  $p\_done = FALSE \Rightarrow PUMPP = PUMP$

$phase=2 \wedge$

*inv1\_7* :  $p\_done = TRUE \wedge change = FALSE \Rightarrow PUMPP = PUMP$

*inv1\_8* :  $p\_done = TRUE \wedge change = TRUE \Rightarrow PUMPP \neq PUMP$

*inv1\_9* :  $p\_done = TRUE \wedge change = FALSE \wedge PUMPP = on \Rightarrow LP = L + v1 * t\_act$

*inv1\_10* :  $p\_done = TRUE \wedge change = FALSE \wedge PUMPP = off \Rightarrow LP = L - v2 * t\_act$

*inv1\_11* :  $p\_done = TRUE \wedge change = TRUE \wedge PUMPP = on \Rightarrow LP = L - v2 * t\_act$

*inv1\_12* :  $p\_done = TRUE \wedge change = TRUE \wedge PUMPP = off \Rightarrow LP = L + v1 * t\_act$

**EVENTS**

**Initialisation**

**begin**

*act2* :  $pump := PUMPi$

*act4* :  $phase := 1$

*act5* :  $LP := Li$

*act6* :  $PUMPP := PUMPi$

*act7* :  $p\_done := FALSE$

*act8* :  $change := FALSE$

**end**

**Event** *decide\_1*  $\hat{=}$

Same as in abstraction

**refines** *decide\_1*

**when**

*grd1* :  $phase = 1$

*grd2* :  $pump = on$

*grd3* :  $LP + v1 * t\_sen \leq LM - v1 * t\_act$

```
    then
      act1 : phase := 2
    end
Event decide_2  $\hat{=}$ 
  Same as in abstraction
refines decide_2
  when
    grd1 : phase = 1
    grd2 : pump = on
    grd3 :  $LP + v1 * t\_sen > LM - v1 * t\_act$ 
      The water level cannot continue to go up.
      It will be dangerous (too high) at next step
      in t_sen seconds.
  then
    act1 : pump := off
    act2 : phase := 2
  end
Event decide_3  $\hat{=}$ 
  Same as in abstraction
refines decide_3
  when
    grd1 : phase = 1
    grd2 : pump = off
    grd3 :  $LP - v2 * t\_sen \geq Lm + v2 * t\_act$ 
      The water level can continue to go down.
      It will not be dangerous at next step in
      t_sen seconds.
  then
    act1 : phase := 2
  end
Event decide_4  $\hat{=}$ 
  Same as in abstraction
refines decide_4
  when
    grd1 : phase = 1
    grd2 : pump = off
    grd3 :  $LP - v2 * t\_sen < Lm + v2 * t\_act$ 
      The water level cannot continue to go down.
      It will be dangerous (too low) at next step
      in t_sen seconds.
  then
    act1 : pump := on
    act2 : phase := 2
  end
Event pump_off  $\hat{=}$ 
  Possible modification of the pump when it is off
```

```

when
    grd1 : phase = 2
    grd2 : PUMPP = off
    grd3 : p_done = FALSE
then
    act1 : LP := LP - v2 * t_act
           Water level continues to go down at speed v2
           for t_act seconds
    act2 : PUMPP := pump
    act3 : p_done := TRUE
    act4 : change := bool(pump = on)
end
Event pump_on ≐
           Possible modification of the pump when it is on
when
    grd1 : phase = 2
    grd2 : PUMPP = on
    grd3 : p_done = FALSE
then
    act1 : LP := LP + v1 * t_act
           Water level continues to go up at speed v1
           for t_act seconds
    act2 : PUMPP := pump
    act3 : p_done := TRUE
    act4 : change := bool(pump = off)
end
Event env_1 ≐
           Sending the water level when pump is on and no change
refines env_1
when
    grd1 : phase = 2
    grd2 : PUMPP = on
    grd4 : p_done = TRUE
    grd5 : change = FALSE
then
    act1 : LP := LP + v1 * t_sen - v1 * t_act
    act2 : phase := 1
    act3 : p_done := FALSE
end
Event env_2 ≐
           Sending the water level when pump is off and no change
refines env_2
when
    grd1 : phase = 2
    grd2 : PUMPP = off
    grd3 : p_done = TRUE

```

```

    then grd4 : change = FALSE
  then
    act1 : LP := LP - v2 * t_sen + v2 * t_act
    act2 : phase := 1
    act3 : p_done := FALSE
  end
Event env_3 ≐
  Sending the water level when pump is off and change
refines env_3
  when
    grd1 : phase = 2
    grd2 : PUMPP = off
    grd3 : p_done = TRUE
    grd4 : change = TRUE
  then
    act1 : LP := LP - v2 * t_sen + v2 * t_act
    act2 : phase := 1
    act3 : p_done := FALSE
  end
Event env_4 ≐
  Sending the water level when pump is on and change
refines env_4
  when
    grd1 : phase = 2
    grd2 : PUMPP = on
    grd3 : p_done = TRUE
    grd4 : change = TRUE
  then
    act1 : LP := LP + v1 * t_sen - v1 * t_act
    act2 : phase := 1
    act3 : p_done := FALSE
  end
END

```

### 5.2.3 m2

An Event-B Specification of w\_m2  
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```

MACHINE w_m2
  Technical refinement for merging env1 and env4
  and also env2 and env3
REFINES w_m1
SEES w_c0
VARIABLES

```

*pump*

---

```

    phase
    LP
    PUMPP
    p_done
EVENTS
Initialisation
    begin
        act2 : pump := PUMPi
        act4 : phase := 1
        act5 : LP := Li
        act6 : PUMPP := PUMPi
        act7 : p_done := FALSE
    end
Event decide_1  $\hat{=}$ 
    Same as in abstraction
extends decide_1
    when
        grd1 : phase = 1
        grd2 : pump = on
        grd3 :  $LP + v1 * t\_sen \leq LM - v1 * t\_act$ 
    then
        act1 : phase := 2
    end
Event decide_2  $\hat{=}$ 
    Same as in abstraction
extends decide_2
    when
        grd1 : phase = 1
        grd2 : pump = on
        grd3 :  $LP + v1 * t\_sen > LM - v1 * t\_act$ 
        The water level cannot continue to go up.
        It will be dangerous (too high) at next step
        in t_sen seconds.
    then
        act1 : pump := off
        act2 : phase := 2
    end
Event decide_3  $\hat{=}$ 
    Same as in abstraction
extends decide_3
    when
        grd1 : phase = 1
        grd2 : pump = off

```

---

```

    grd3 :  $LP - v2 * t\_sen \geq Lm + v2 * t\_act$ 
           The water level can continue to go down.
           It will not be dangerous at next step in
           t_sen seconds.
  then
    act1 :  $phase := 2$ 
  end
Event decide_4  $\hat{=}$ 
  Same as in abstraction
extends decide_4
  when
    grd1 :  $phase = 1$ 
    grd2 :  $pump = off$ 
    grd3 :  $LP - v2 * t\_sen < Lm + v2 * t\_act$ 
           The water level cannot continue to go down.
           It will be dangerous (too low) at next step
           in t_sen seconds.
  then
    act1 :  $pump := on$ 
    act2 :  $phase := 2$ 
  end
Event pump_on  $\hat{=}$ 
refines pump_on
  when
    grd1 :  $phase = 2$ 
    grd2 :  $PUMPP = on$ 
    grd3 :  $p\_done = FALSE$ 
  then
    act1 :  $LP := LP + v1 * t\_act$ 
    act2 :  $PUMPP := pump$ 
    act3 :  $p\_done := TRUE$ 
  end
Event pump_off  $\hat{=}$ 
refines pump_off
  when
    grd1 :  $phase = 2$ 
    grd2 :  $PUMPP = off$ 
    grd3 :  $p\_done = FALSE$ 
  then
    act1 :  $LP := LP - v2 * t\_act$ 
    act2 :  $PUMPP := pump$ 
    act3 :  $p\_done := TRUE$ 
  end
Event sensor_on  $\hat{=}$ 
refines env_1, env_4

```

```
when
  grd1 : phase = 2
  grd2 : PUMPP = on
  grd3 : p_done = TRUE
then
  act1 : LP := LP + v1 * t_sen - v1 * t_act
  act2 : phase := 1
  act3 : p_done := FALSE
end
Event sensor_off  $\hat{=}$ 
refines env_2, env_3
when
  grd1 : phase = 2
  grd2 : PUMPP = off
  grd3 : p_done = TRUE
then
  act1 : LP := LP - v2 * t_sen + v2 * t_act
  act2 : phase := 1
  act3 : p_done := FALSE
end
END
```



